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Behavior of Integrate-and-Fire Model with Reversal Potentials Driven by Low Correlation Renewal Inputs

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Abstract

The integrate-and-fire model with reversal potentials driven by low correlation renewal inputs is studied. we firstly propose two novel approximation schemes for integrate-and-fire(IF) model with renewal process inputs. Then, we consider how Low correlation renewal inputs affect the integrate-and-fire model with reversal potentials. For low positive, mean firing time is a decreasing function of input correlation. Interestingly, for low negative correlations, mean firing time is nondecreasing of input correlation.

Mathematics Subject Classification: 60K30, 65C40

Keywords: Integrate-and-Fire model, Renewal process, The interspike interval, Correlation

1 Introduction

Although the single neuron model with random inputs has been widely studied in theory and in computo, most such studies are done under the assumption that inputs are poisson processes[1,2,3]. This assumption is certainly an approximation to the physiological data, Because, at least for the IF model, its output spike trains are renewal processes rather than poisson processes. We study behavior of integrate-and-fire model with reversal potentials driven by low correlation renewal inputs.

However, it is very difficult theoretically to deal with a system with renewal form inputs(even with poissonian form inputs). In the literature,many papers have been devoted to the issue discussed here for poisson process inputs. In particular, the usual approximate scheme(UAS) was proposed early in the literature to approximate poisson process inputs. The idea is simple: to find a diffusion process which has the same mean and variance as the poisson process. We apply the similar idea to renewal process inputs. By omitting higher order terms in the variance of a renewal process, a new approximation scheme is proposed in the current paper.

2 Approximation

For the simplicity of notation, we first consider the case of single renewal process. Let $T_1, T_2, T_3, ...,$ be series of the time between events(spikes), and which are independently and identically distributed random variables, using T denoting them. Let $f(t)$ be the probability density function of random variable T. We assume that the mean, variance and third central moment of T exist and denote them by λ, α^2 and λ_3 respectively. Let $\{N_t : t \geq 0\}$ be the corresponding renewal process. The expected number and variance of events in $(0, t]$ are(see[4]) P_{556} and [5] P_{72})

$$\mu(t) \equiv < N_t >= \frac{t}{\lambda} + \frac{\alpha^2 - \lambda^2}{2\lambda^2}, \sigma^2(t) \equiv < (N_t)^2 > - < N_t >^2 . \tag{1}$$

A useful general result for renewal processes is that(see[5])

$$\sigma^2(t) = \frac{\alpha^2}{\lambda^3} t + (\frac{1}{6} + \frac{\alpha^4}{2\lambda^4} - \frac{\lambda_3}{3\lambda^3}) + 0(1) = \frac{\alpha^2}{\lambda^3}[t + (\frac{\lambda^3}{6\alpha^2} + \frac{\alpha^2}{2\lambda} - \frac{\lambda_3}{3\alpha^2})] + 0(1). \tag{2}$$

From Eqs (1)and(2), we get

Theorem 2.1 *The usual approximation takes the following form*

$$\begin{cases} dN_t \sim \frac{1}{\lambda} dt + \frac{\alpha}{\lambda^{3/2}} dB_t \\ B_0 = \frac{\alpha^2 - \lambda^2}{2\lambda^2}. \end{cases}$$

where$B(t)$is the standard brownian motion. We call the approximation in Theorem 2.1 usual approximate scheme(UAS).

Looking at the variance $\sigma^2(t)$, we see that the leading term we omit in the UAS is $\frac{1}{6} + \frac{\alpha^4}{2\lambda^4} - \frac{\lambda_3}{3\lambda^3}$. Since in the UAS only the derivative of $\sigma^2(t)$ is used, the constant term disappears. We therefore want to find a process $\eta^{\alpha,\lambda}(t)$ satisfying the property such that both its first and second order moment are in agreement with the first and second order moment of the renewal process. In other words, we want to find a process $\eta^{\alpha,\lambda}(t)$ with

$$< (B(t) - \eta^{\alpha,\lambda}(t))^2 >= t + (\frac{\lambda^3}{6\alpha^2} + \frac{\alpha^2}{2\lambda} - \frac{\lambda_3}{3\alpha^2}).$$

Then we use the process $d\tilde{N}_t \sim \frac{1}{\lambda}dt + \frac{\alpha}{\lambda^{3/2}}d(B_t - \eta^{\alpha,\lambda}(t))$. approximating the renewal process N(t). we choose an Ornstein-Uhlenbeck(OU) process

$$\begin{cases} d\xi_t = -b\xi_t dt + dB_t \\ \xi_0 = 0. \end{cases}$$

where $b > 0$ is a constant. Let $f(t, x) = e^{-bt}x$, by the Ito rule, we have $\xi_t = \int_0^t exp[-b(t-s)]dB_s$. Let $\eta^{\alpha,\lambda}(t) = c\xi_t = c\int_0^t exp[-b(t-s)]dB_s$, by $<(B_t - \eta^{\alpha,\lambda}(t))^2> = t + (\frac{\lambda^3}{6\alpha^2} + \frac{\alpha^2}{2\lambda} - \frac{\lambda_3}{3\alpha^2})$, thus $c = c_r \equiv 2 - \sqrt{4 + b(\frac{\lambda^3}{3\alpha^2} + \frac{\alpha^2}{\lambda} - \frac{2\lambda_3}{3\alpha^2})}$. Hence we find a new scheme to approximate N(t).

Theorem 2.2 *The renewal processes approximation takes the following form*

$$d\tilde{N}_t = \frac{1}{\lambda}dt + \frac{\alpha}{\lambda^{3/2}}d[B_t - c_r d\xi(t))].$$

where $c_r = 2 - \sqrt{4 + b(\frac{\lambda^3}{3\alpha^2} + \frac{\alpha^2}{\lambda} - \frac{2\lambda_3}{3\alpha^2})}, \xi(t)$ *satisfying*

$$\begin{cases} d\xi_t = -b\xi_t dt + dB_t \\ \xi_0 = 0. \end{cases}$$

We call the approximation in Theorem 2.2 Ornstein-Uhlenbeck scheme(OUS).

3 Model

For two given quantities V_{th}(threshold)$>$ V_{rest}(resting potential), and when $V_t < V_{th}$, the membrane potential V_t satisfies the following dynamics:

$$\begin{cases} dV_t = -\frac{1}{\gamma}(V_t - V_{rest})dt + a(V_E - V_t)\sum_{i=1}^p dN_i^E(t) + b(V_I - V_t)\sum_{j=1}^q dN_j^I(t) \\ V_0 = V_{rest} = B_0. \end{cases} \tag{3}$$

where $\gamma > 0$ and $\frac{1}{\gamma}$ is the decay rate, $a > 0$ is the magnitude of EPSPS(excitatory postsynaptic potentials), $b > 0$ is he magnitude of IPSPS(inhibitory postsynaptic potentials), and V_E and V_E are the reversal potentials. V_t is now a birth-and-death process with boundaries V_E and V_E. $N_i^E(t)$ and $N_j^I(t)$ are renewal process (EPSPS and IPSPS). Once V_t is greater than V_{th}, a spike is generated, and it is reset to $V_{rest} = B_0$. This model is termed the Integrate-and-Fire model with reversal potentials(see[6,7]). We define $T(\lambda, \gamma, r, c) = inf\ (t > 0, V_t = V_{th})$ as the firing time(interspike intervals).
We apply usual approximate scheme to approximate the inputs of the Integrate-and-Fire model

$$dN_i^E(t) \sim \frac{1}{\lambda}dt + \frac{\alpha}{\lambda^{3/2}}dB_i^E(t), dN_j^I(t) \sim \frac{1}{\lambda}dt + \frac{\alpha}{\lambda^{3/2}}dB_j^I(t). \tag{4}$$

where $B_i^E(t), B_j^I(t)$ is the standard Brownian motions. Without loss of generality, we assume that the correlation coefficient between ith excitatory(inhibitory) synapse and jth excitatory(inhibitory) synapse is

$$\rho_E(i,j) = \frac{cov(N_i^E(t), N_j^E(t))}{\sqrt{var(N_i^E(t))var(N_j^E(t))}} = \frac{cov(N_i^E(t), N_j^E(t))}{\sigma^2(t)}.$$

by (4)

$$\begin{aligned} cov(N_i^E(t), N_j^E(t)) &= cov(\frac{1}{\lambda}t + \frac{\alpha}{\lambda^{3/2}}B_i^E(t), \frac{1}{\lambda}t + \frac{\alpha}{\lambda^{3/2}}B_j^I(t)) \\ &= \frac{\alpha^2}{\lambda^3}cov[B_i^E(t), B_j^E(t)]. \end{aligned}$$

Let $c_E(i,j) = cov(B_i^E(t), B_j^E(t))$,have $c_E(i,j) = \rho_E(i,j)\sigma^2(t) = \rho_E(i,j)[t + (\frac{\lambda^3}{6\alpha^2} + \frac{\alpha^2}{2\lambda} - \frac{\lambda_3}{3\alpha^2})]$. as the same reason $c_I(i,j) = \rho_E(i,j)\sigma^2(t) = \rho_I(i,j)[t + (\frac{\lambda^3}{6\alpha^2} + \frac{\alpha^2}{2\lambda} - \frac{\lambda_3}{3\alpha^2})]$.

Since the summation of Brownian motions is a Brownian motion, thus

$$\begin{aligned} \sigma^2 &= \frac{(a(V_E - V_t))^2\alpha^2}{\lambda^3}p + \frac{(b(V_I - V_t))^2\alpha^2}{\lambda^3}q \\ &+ \frac{(a(V_E - V_t))^2\alpha^2}{\lambda^3}\sum_{i\neq j}^{p} c_E(i,j) + \frac{(b(V_I - V_t))^2\alpha^2}{\lambda^3}\sum_{i\neq j}^{q} c_I(i,j). \end{aligned}$$

$$\mu = \frac{a(V_E - V_t)p - b(V_I - V_t)q}{\lambda} \tag{5}$$

4 Numerical Results

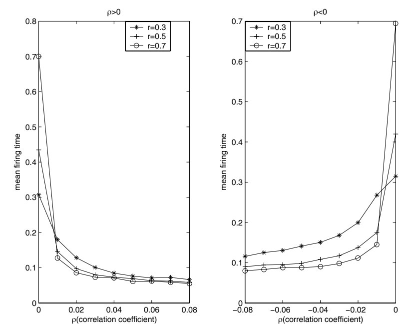
To actually carry out numerical simulations, we have to further confine ourselves to the case that T is distributed according to a Gamma distribution, the corresponding density function is $f(t) = (\frac{1}{\mu})^\kappa \frac{t^{\kappa-1}e^{-(t/\mu)}}{\Gamma(\kappa)}$. Then we have $< T >= \kappa\mu, var < T >= \kappa\mu^2, < (T - \kappa\mu)^3 >= 2\kappa\mu^3$, Thus we have that

$$\lambda = \kappa\mu, \alpha^2 = \kappa\mu^2, \lambda_3 = 2\kappa\mu^3.$$

For the simplicity of notation,we assume that $p = q, b = ar, \rho_E(i,j) = \rho_I(i,j) = \rho, c_E(i,j) = c_I(i,j) = c$. thus(3) becomes

$$\begin{cases} dV_t = -\frac{1}{\gamma}(V_t - V_{rest})dt + \frac{a(V_E - V_t)p - b(V_I - V_t)q}{\kappa\mu}dt + \\ \sqrt{\frac{(a(V_E - V_t))^2}{\kappa^2\mu}(p + c(p^2 - p)) + \frac{(b(V_I - V_t))^2}{\kappa^2\mu}(q + c(q^2 - q))}dB(t) \\ V_0 = V_{rest} = B_0. \end{cases} \tag{6}$$

We employ the following parameters in our numerical simulations in matlab7.0: $\kappa = 0.5, \mu = 10, \gamma = 20.2msec, p = 100, q = pr, v_{th} = 20mv, v_{rest} = B_0, a(V_E - V_t) = a(V_E - V_t) = 1mv$, which is the set of parameters used elsewhere(see[7]) We numerically solve the membrane equations of V_t in Eq.(6) with a step size of 0.001. In simulation, 1000 spikes are generated for each set of parameters. V_t is reset to $v_{rest} = B_0$.



For low positive correlations, mean firing time is a decreasing function of input correlation. For low negative correlations, mean firing time is nondecreasing of input correlation.

5 Discussion

Neurons emit and receive spike trains of non-poisson perse. How can we approximate the non-poisson spike trains by diffusion type processes? Under the assumption that is a renewal process we provide answers to the question, and two novel approximation schemes are proposed: the usual approximate scheme and the Ornstein-Uhlenbeck process approximate scheme. Then we discuss low correlation between renewal process synaptic inputs impacts on the output of the integrate-and-fire model with reversal potentials. By numerical simulation we see even a weak correlation have a substantial impact on output of Integrate-and-Fire model with reversal potentials.

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